A Weibull Distributed Ameliorating Inventory Model with Ramp Type Demand and Shortages

Dr. Biswaranjan Mandal*

Abstract:

Most of the inventory models dealt with deteriorating items. On the contrary, just few papers considered inventory models under amelioration environment. The present paper deals with an inventory model under the environment of Amelioration following Weibull Distribution. The demand rate is considered as a combination of a linear and quadratic function of time followed by Ramp type. It becomes quadratic function of time in the beginning of the cycle, and linear as passage of time. Shortages are allowed and which is fully backlogged. The problem is to minimize the total inventory cost. The developed model is illustrated by a numerical example and finally the effect of amelioration on the model is discussed by its sensitivity analysis.

Key Words: *Inventory, Weibull distribution, ramp type, quadratic, linear, shortages, ameliorating items, shape and scale parameter, inventory cost, fully backlogged, sensitivity analysis etc.*

Subject classification: *AMS (American Mathematical Society) Classification No. 90B05*

1. Introduction:

In recent trends businessmen have shown an increasing awareness of the need for precision in the field of inventory control of deteriorating items. As a result, many deterioration models have been consequently developed. Yet we have not observed much appreciation of ameliorating consideration. Due to lack of considering the influence of demand, the ameliorating items assuming duration for the amount of inventory will gradually increase; meanwhile, in the traditional inventory model dealing with deteriorating items the amount of inventory will gradually decrease. Amid the published literatures, scholars and researchers do not pay much attention to the ameliorating problems and items. To cope with this deficiency, lately a few studies are concerned with the problems of amelioration, because they do exist in the real world such as the farming, fishery, and poultry industries. The fast-growing animals like ducks, pigs, and broilers in poultry farms, highbred fishes in ponds, and the cultivation of vegetables and fruits in farms are typical field applications. This is quite different from the deteriorating items and deserves a comprehensive study. *Hwang[1], Mondal[2], Tuan-et-al.[3]* developed the inventory

**Associate Professor of Mathematics;* Acharya Jagadish Chandra Bose College, Kolkata; *Email :* drbrmajcbc@gmail.com

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models with ameliorating items under various situations. Hwang is one of first authors to consider the remodeling from deterioration to amelioration. However, his published literatures did not present the ultimately optimal solution. Instead, he used a graphical procedure of *Gupta*[4] to estimate the optimal cycle time, which could not derive to a convincingly optimal solution. In this paper we developed an inventory model under the environment of Amelioration followed by Weibull Distribution where this distribution has been applied to the problems because it can describe the different life spans effectively by utilizing the changes of parameters.

Since, inventory is an important part of our manufacturing, distribution and retail infrastructure where demand plays an important role in choosing the best inventory policy. Many researchers like *Jalan-et-al*[5], *Giri-et-al*[6], *Mandal-and-Pal*[7], *Biswaranjan-Mandal*[8], *Datta-&-Pal*[9] etc. were engaged to develop the inventory models assuming the demand of the items to be constant, linear trended, exponential increasing or decreasing, power demand pattern, alternating demand with time. Later we observed that such type of demands do not precisely depict the demand of certain items such as newly launched fashion items, garments, cosmetics, automobiles, mobiles, computers etc. for which the demand increases as launched the items into the market, and after some times, it becomes constant. This ramp type demand is developed by *Mishra-&-Singh*[10], *Mandal-&-Pal*[11], *Agarwal.et.al*[12], *Singh.et.al.*[13] and many other researchers. Here we discussed the ramp type demand which is combination of a linear and quadratic function of time. For this sort of situation, an effort has been made to analyze an inventory model for Weibull distributed ameliorating items assuming quite realistic demand rate as combination of linear and quadratic function of time. Shortages are allowed which are fully backlogged. Finally, the developed model is illustrated by a numerical example and also the effect of amelioration on the model is furnished with the help of its sensitivity analysis.

2. Assumptions and Notations:

The inventory model is developed under the following assumptions and notations:

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(i) Replenishment size is constant and replenishment rate is infinite.
(ii) Lead time is zero.
(iii) T is the fixed length of each production cycle.
(iv) C_0 is the ordering cost/order.
(v) C_p is purchase cost per unit with $C_p < C_a$.
(vi) C_h is the inventory holding cost per unit.
(vii) C_a is the amelioration cost per unit.
(viii) C_s is the shortage cost per unit.
(ix) I(t) is the inventory level.
(x) TC is the average total cost per unit time.
(xi) A(t) is the amelioration rate following Weibull distributed

$A(t) = \alpha\beta t^{\beta-1}, 0 \le \alpha << 1, \beta \ge 1$,

Where, α is the shape parameter and β is the scale parameter.

(xii) The ramp type demand rate R(t) is a combination of linear and quadratic function of time defined as follows

R(t) = a + bt + $c\{t-(t-\mu)H(t-\mu)\}t$ where $H(t-\mu)$ is the Heaviside's function defined as follows

$$H(t-\mu) = \begin{cases} 0, t < \mu \\ 1, t \ge \mu \end{cases}$$ and a, b, c >0

Thus, demand can be written as R(t) = $\begin{cases} a+bt+ct^2, t < \mu \\ a+kt, t \ge \mu \end{cases}$

Where, k=$b+c\mu$.

(x). Shortages are allowed which are fully backlogged.

3. Formulation and Solution of the Model:

Let Q be the on-hand inventory produced or purchased at the beginning of each period. The differential equations which the on-hand inventory I(t) must satisfy during the cycle time T is the following-

$$\frac{dI(t)}{dt} - A(t)I(t) = -R(t), 0 \le t \le t_1 \quad (3.1)$$

And $$\frac{dI(t)}{dt} = -R(t), t_1 \le t \le T \quad (3.2)$$

In this model, we assume $\mu < t_1$ and therefore the above governing equations become

$$\frac{dI(t)}{dt} - \alpha\beta t^{\beta-1} I(t) = -(a+bt+ct^2), 0 \le t \le \mu \quad (3.3)$$

$$\frac{dI(t)}{dt} - \alpha\beta t^{\beta-1} I(t) = -(a+kt), \mu \le t \le t_1 \quad (3.4)$$

And $\frac{dI(t)}{dt} = -(a+kt), t_1 \le t \le T$ (3.5)

The initial condition is $I(0) = Q$ and $I(t_1) = 0$ (3.6)

Solutions of the equations (3.3), (3.4) and (3.5) using (3.6) and neglecting second and higher powers of α (<<1) (as $O(\alpha^2)$ are very small) are the following

$$I(t) = Q(1+\alpha t^{\beta}) - \{at + \frac{b}{2}t^2 + \frac{c}{3}t^3 + \frac{a\alpha\beta}{\beta+1}t^{\beta+1} + \frac{b\alpha\beta}{2(\beta+2)}t^{\beta+2} + \frac{c\alpha\beta}{3(\beta+3)}t^{\beta+3}\}, 0 \le t \le \mu \quad (3.7)$$

$$I(t) = a(t_1 - t) + \frac{k}{2}(t_1^2 - t^2) - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{a\alpha\beta}{\beta+1}t^{\beta+1}$$

$$-\frac{k\alpha}{\beta+2}t_1^{\beta+2} - \frac{k\alpha\beta}{2(\beta+2)}t^{\beta+2} + a\alpha t_1 t^{\beta} + \frac{k\alpha}{2}t_1^2 t^{\beta}, \mu \le t \le t_1 \quad (3.8)$$

And $I(t) = a(t_1 - t) + \frac{k}{2}(t_1^2 - t^2), t_1 \le t \le T$ (3.9)

Equating the expressions (3.7) and (3.8) at $t = \mu$ and neglecting second and higher powers of α (<<1) as $O(\alpha^2)$ are very small, we get

$$Q = at_1 + \frac{k}{2}t_1^2 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{k\alpha}{\beta+2}t_1^{\beta+2} \quad (3.10)$$

4. Cost Components:

Total Cost: The total cost during the period [0, T] comprises of the sum of the ordering cost, purchase cost, holding cost, ameliorating cost and shortage cost.

1. Total ordering cost **(OC)** during the period [0, T] = C_o (fixed)
2. Total purchase cost **(PC)** during the period [0, T] = C_p I(0) = C_p Q

$$= C_p \{at_1 + \frac{k}{2}t_1^2 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{k\alpha}{\beta+2}t_1^{\beta+2}\}$$

3. Total Holding cost **(HC)** during the period [0, T] = $C_h \int_0^{t_1} I(t)dt$ = $C_h[\int_0^{\mu} I(t)dt + \int_{\mu}^{t_1} I(t)dt]$

$$= C_h[\frac{a}{2}t_1^2 + \frac{k}{3}t_1^3 - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{k\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3} + \frac{c}{12}\mu^4 + \frac{c\alpha\beta(\beta+8)}{6(\beta+2)(\beta+3)(\beta+4)}\mu^{\beta+4}]$$

4. Total Amelioration cost **(AMC)** during the period [0, T] = $C_a \int_0^{t_1} \alpha\beta t^{\beta-1} I(t)dt$

$$= C_a \alpha\beta[\int_0^{\mu} t^{\beta-1}I(t)dt + \int_{\mu}^{t_1} t^{\beta-1}I(t)dt]$$

$$= C_a[\frac{a\alpha}{\beta+1}t_1^{\beta+1} + \frac{k\alpha}{\beta+2}t_1^{\beta+2} + \frac{a\alpha\beta}{\beta+1}\mu^{\beta+1} + \frac{k\alpha\beta}{2(\beta+2)}\mu^{\beta+2}]$$

5. Total Shortage cost **(SC)** during the period [0, T] = $-C_s \int_{t_1}^{T} I(t)dt$

$$= C_s[\frac{a}{2}(T-t_1)^2 + \frac{k}{6}(T^3 - 3Tt_1^2 + 2t_1^3)]$$

Therefore, the average total cost per unit time over the cycle [0, T] is given by

$$TC\,(t_1) = \frac{1}{T}[OC + PC + HC + AMC + SC]$$

$$= \frac{C_o}{T} + \frac{C_p}{T}[at_1 + \frac{k}{2}t_1^2 - \frac{a\alpha}{\beta+1}t_1^{\beta+1} - \frac{k\alpha}{\beta+2}t_1^{\beta+2}] +$$

$$\frac{C_h}{T}[\frac{a}{2}t_1^2 + \frac{k}{3}t_1^3 - \frac{a\alpha\beta}{(\beta+1)(\beta+2)}t_1^{\beta+2} - \frac{k\alpha\beta}{(\beta+1)(\beta+3)}t_1^{\beta+3} + \frac{c}{12}\mu^4 + \frac{c\alpha\beta}{6(\beta+2)(\beta+3)(\beta+4)}\mu^{\beta+4}]$$

$$+ \frac{C_a}{T}[\frac{a\alpha}{\beta+1}t_1^{\beta+1} + \frac{k\alpha}{\beta+2}t_1^{\beta+2} + \frac{a\alpha\beta}{\beta+1}\mu^{\beta+1} + \frac{k\alpha\beta}{2(\beta+2)}\mu^{\beta+2}]$$

$$+ \frac{C_s}{T}[\frac{a}{2}(T-t_1)^2 + \frac{k}{6}(T^3 - 3Tt_1^2 + 2t_1^3)] \qquad (4.1)$$

For minimum, the necessary condition is $\frac{dTC}{dt_1} = 0$

Or, $C_p + (C_a - C_p)\alpha t_1^{\beta} + C_h(t_1 - \frac{\alpha\beta}{\beta+1}t_1^{\beta+1}) - C_s(T-t_1) = 0$ (4.2)

which is the equation for optimum solution.

Let t_1^* be the positive real root of the above equation (4.2), then optimal values Q^* of Q and TC^* of TC are obtained by putting the value $t_1 = t_1^*$ in the expressions (3.10) and (4.1).

Note: If there be no amelioration environment i.e. α = 0, the equation (4.2) becomes

$$C_p + C_h t_1 - C_s(T-t_1) = 0 \text{ or } t_1 = \frac{C_s T - C_p}{C_h + C_s}$$

5. Numerical Example:

To illustrate the model numerically, we use the following parameter values:

C_o = \$200; C_p = \$5; C_h = \$12; C_a = \$7; C_s = \$15; α = 0.001; β = 2; a =30; b = 6; c = 5;

μ = 0.12 year; T = 1 year.

Solving the quadratic equation (4.2) with the above numerical values, we find the optimum values of t_1 as t_1^* = 0.37 year.

The optimum values Q^*, purchase cost (PC^*), holding cost (HC^*), amelioration cost (AMC^*) and shortage cost(SC^*) are Q^* = 11.53 units, PC^* = Rs. 57.66, HC^* = Rs26.03, AMC^* = Rs 0.22 and SC^* = Rs 100.58.

The minimum average total cost per unit time is found to be TC^* = Rs 384.50.

6. Sensitivity Analysis:

We now study the effects of amelioration on the model by putting different the values of shape parameter (α) and scale parameter (β). The results of this analysis are shown in the following tables.

Table A: Sensitivity study of the shape parameter (α).

β	α	Optimum values of					
		Q	Purchase cost	Holding cost	Amelioration cost	Shortage cost	Total cost
2	0.001	11.5324	57.662	26.033	0.221	100.583	384.499
	0.005	11.5310	57.655	26.031	0.236	100.576	384.499
	0.010	11.5292	57.646	26.029	0.256	100.569	384.499
	0.015	11.5275	57.638	26.026	0.275	100.561	384.500
	0.020	11.5257	57.629	26.024	0.294	100.554	384.500
	0.025*	11.5224	57.612	26.021	0.313	100.546	**384.492**
	0.030	11.5222	57.611	26.019	0.332	100.539	384.501
	0.035	11.5204	57.602	26.016	0.351	100.532	384.501
	0.040	11.5187	57.594	26.014	0.370	100.524	384.502
	0.045	11.5169	57.585	26.011	0.388	100.516	384.502
	0.050	11.5152	57.576	26.009	0.408	100.509	384.502

Table B: Sensitivity study of the scale parameter (β).

α	β	Optimum values of					
		Q	Purchase cost	Holding cost	Amelioration cost	Shortage cost	Total cost
0.001	1 (constant amelioration rate)	11.4501	57.251	26.031	0.798	100.583	384.663
	2	11.5324	57.662	26.033	0.221	100.583	384.499
	3	11.5545	57.773	26.034	0.065	100.583	384.455
	4	11.5609	57.805	26.034	0.020	100.584	384.443
	5	11.5629	57.815	26.034	0.006	100.584	384.439
	6	11.5635	57.818	26.034	0.002	100.584	384.438
	7	11.5637	57.819	26.034	0.0007	100.584	384.438
	8	11.5637	57.819	26.034	0.0002	100.584	384.437
	9	11.5638	57.819	26.034	0.00008	100.584	384.437
	10	11.5638	57.819	26.034	0.00003	100.584	384.437

Analyzing the results given in the above Table A and Table B, the following observations are made-

(i) If the shape parameter (α) increases, the optimum value of amelioration cost increases , whereas optimum values of Q, Purchase cost , Holding cost and Shortage cost decrease with increase of α. But it is observed that as α increases, the optimum value of the total cost (TC) decreases and increases simultaneously, and it becomes minimum at α = 0.025.

(ii) When the scale parameter (β) increases, the optimum values Q, Purchase cost, Holding cost and Shortage cost increase, whereas amelioration cost decreases with increase of β. But it is observed that the total cost (TC) decreases very insensitively towards the changes of β.

On the above observation, it can be concluded that the shape and scale amelioration parameters play an important role on the estimation of optimal cost of the present inventory model and so we need more attention to estimate these parameters. The application concerned with the problems of amelioration exists in the real world such as the farming, fishery, and poultry industries. The fast-growing animals like ducks, pigs, and broilers in poultry farms, highbred fishes in ponds, and the cultivation of vegetables and fruits in farms are typical field applications. This is quite different from the deteriorating items and deserves a comprehensive study.

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