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Evidence of Decoupling in Bitcoin-Stock Market Price Relationship

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Abstract:

Cryptocurrency has gained widespread acceptance among speculators and investors alike owing to its volatility and its promised utility. With a burgeoning crypto market cap, dominated by Bitcoin, it is of paramount importance to identify and establish the relationship between an evolving Bitcoin and the backbone of traditional economic system - Stock market. This paper focuses on (a) Establishing instability and finding structural breaks (b) Identifying causal relationships between Bitcoin and S&P 500 (c)Using time series analysis to hypothesize decoupling of Bitcoin and stock market. The time series uses Daily prices, dating back to 2014. For the purposes of identifying structural breaks - CUSUM and Bai-Perron tests were used. Empirical relationship between Bitcoin and S&P 500 were identified using Granger full sample causality and Johannsen's Cointegration test. Impulse response tests were performed. Finally, DCC-GARCH setup was used for measuring spill overs in volatility. The paper finds break dates at 2017:10 and 2020:9. Through establishing the linkages, or lack thereof, the study tracked the development of Bitcoin's relationship with traditional equity market, concluding with a finding of statistically significant evidences of decoupling across periods. Furthermore, causes for conjectures and suggestions for future studies were stated.

Keywords: *Structural Breaks, Decoupling, Cointegration, DCC-GARCH, CUSUM, Granger-Causality, Bai-Perron Multiple Breakpoint Test, ARDL*

1. Introduction:

Bitcoin originations lie in the white paper that was published in 2008 under the pseudonym "Satoshi Nakamoto". Aiming to disrupt the traditional financial systems. Bitcoin was envisioned as a new-age alternative to Gold and Currency.

Often compared to digital gold, Bitcoin is heralded as an even better store of value than its metal counterpart. On the other hand, its blockchain authentication and deflationary nature has time and again led to comparing it with 'Government Money' - Fiat currency. Owing to its varied utility, both as a standard of exchange and as a store of value, Bitcoin has garnered widespread interest from its users and speculators alike. More recently, it has attracted the attention of institutional and retail investors - who have been attracted by its astronomical returns. This can be evidenced by the growth in its market cap. At its peak, Bitcoin by itself was worth a market cap of 1.1 trillion US dollars, which it had achieved in a little over nine years. In addition to individual investors, Crypto has also attracted the attention of fund managers who speculate on higher returns of bitcoin despite the added volatility. Yet, what has soured this seemingly colossal rise in Bitcoin price has been equally massive falls, Elon Musk (2021)[1]. Bitcoin has had explosive rises, and devastating crashes-however, studies haven't focused on whether these co-explosive phenomena were exhibited alongside volatility in traditional stock market.

Thus, the question of whether Bitcoin's falls were arbitrary or in fact caused by disturbances in traditional financial markets, most importantly, the stock markets assume fundamental importance. This relationship is especially important to understand whether bitcoin has reached the critical mass where it has become interlinked the orthodox economic assets. It might also find use among investors and portfolio investors, who may turn to bitcoin to hedge their stock portfolio. It can also provide an insight into investors mentality. Through finding the dynamic correlation between the volatility of 'Bitcoin' and 'Stocks', one can also hypothesize that information flows from the stock to the crypto market but not vice versa. This evidence of information flow can be used for portfolio management purposes. If presence of non-reactivity is found, it can mean that the Bitcoin market is isolated and unaffected by phenomenon affecting stocks. Typically, literature relating to Bitcoin and stock markets are scant to study the relationship over the entirety of Bitcoin's existence.

The novelty of this paper lies in studying the process changes in the Bitcoin - S&P 500 (Standard and Poor's 500) model and then studying the relationship. This enables one to track the journey of the model and through the difference across the periods, making strong assumptions regarding the future evolution of the Bitcoin - S&P 500 relationship. Another insight the paper might provide can be in form of whether the Bitcoin market is becoming self-contained. If such a conjecture can be made, it can be hypothesized that Bitcoin can offer attractive hedging or safe-haven proposition in the near future.

2. Review of Literature:

Cryptocurrency and more specifically – 'Bitcoin' was birthed in the whitepaper published by a programmer, or a group of programmers named Satoshi Nakamoto (2008)[2]. Prior studies on the

[1]Elon Musk (2021). Bitcoin has had 7 crashes where its value fell over 50%. The most recent of the crashes was in May 2021.
[2]Nakamoto, S. (2008). Bitcoin: A Peer-to-Peer Electronic Cash System. Retrieved from https://bitcoin.org/bitcoin.pdf

association of Bitcoin and Stock Market are few and often study the association in its entirety. In totality, the existing literature can be sub-divided into two broad categories – (1) those performing statistical test on Bitcoin and stocks in isolation; and (2) those predicting Bitcoin prices from stock market indices. In particular, the study of Vassiliadis, Papadopoulos, Rangoussi, Konieczny & Gralewski (2017)[3] is of special interest, according to their work, a strong correlation was found between lagged Bitcoin prices and major stock market indices. In the Indian context, the drivers of Bitcoin price were studied by Malik (2020)[4]. However, the study focused on gold prices instead of stock market prices. Notably, stock market did not form part of the model given in Malik (2020). Cointegration of stock market and Bitcoin prices was studied in the Paper of Tekinay & Kocakoc (2018)[5]. Notably, the study used the Engle Granger test to find out the long-term stabilization of the assets, finding no cointegrations between Dow Jones and Bitcoin prices. Further studies on Bitcoins connectedness was studied by Zeng, Yang & Shen (2020)[6] where bitcoin's linkages were studied relative to other financial assets, namely, oil, stocks and gold. A rolling window analysis was followed, finding that bitcoin prices exhibited low degree of connectedness to traditional assets. However, the linkages between Bitcoin and other cryptocurrencies are well documented and plentiful. Tiwari, Adewuyi, Albulescu & Wohar (2020)[7] found significant contagion among price returns of major cryptocurrency. Volatility persistence in Bitcoin was studied by Yaya, Ogbonna, Mudida & Abu (2020)[8] whose study established the presence of higher volatility persistence in the cryptocurrency market.

3. Objectives of the Study:

The paper strives to ascertain the evolution of association between Bitcoin and Stock Market Prices. Its objectives can be defined as follows:

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[3] Vassiliadis, S., Papadopoulos, P., Rangoussi, M., Konieczny, T. and Gralewski, J. (2017). Bitcoin Value Analysis Based On Cross-Correlations; *Journal of Internet Banking and Commerce*, Vol. 22 (S7), January 2017.

[4] Malik, S. (2020). Drivers of Bitcoin Prices: An Empirical Analysis of India, *Journal of Critical Reviews*, Vol. 7 (14), 2020, pp. 1252-1258, ISSN - 2394-5125.

[5] Tekinay, M. and Kocakoç, I.D. (2018). A Study of Relations Between Bitcoin, Currencies, Stock Exchanges and Commodities; *New Trends in Economics and Administrative Sciences, Izmir International Congress on Economic and Administrative Sciences*, December, 2018.

[6] Zeng, T., Yang, M. and Shen, Y. (2020). Fancy Bitcoin and Conventional Financial Assets: Measuring Market Integration Based on Connectedness Networks, *Economic Modelling, Elsevier*, Vol. 90, August 2020, pp. 209-220, ISSN 0264-9993.

[7] Tiwari, A.K., Adewuyi, A.O., Albulescu, C.T. and Wohar, M.E. (2020). Empirical Evidence of Extreme Dependence and Contagion Risk Between Main Cryptocurrencies, *The North American Journal of Economics and Finance*, *Elsevier*, Vol. 51, January 2020. ISSN 1062-9408.

[8] Yaya, O.S., Ogbonna, A.E., Mudida, R. and Abu. N. (2020). Market Efficiency and Volatility Persistence of Cryptocurrency during Pre-and Post-Crash Periods of Bitcoin: Evidence based on Fractional Integration. *International Journal of Finance & Economics. Wiley*, Vol. 26 (2). DOI:10.1002/ijfe.1851

i) Establishing instability in the Bitcoin - S&P 500 model through the CUSUM and CUSUM square test and finding break dates of structural change with Bai-Perron test. Dummy variables were added to remove instability and verify our break dates to be the actual causation of instability.

ii) Testing whether Bitcoin prices and stock markets prices have a long run equilibrium through cointegration test. Finding evidence of cointegration can mean conjunct movement of both prices, thus, diminishing hedging properties in the long run.

iii) Studying whether 'Stock Market Returns' can cause 'Bitcoin returns' and vice versa. This can be used for hypothesizing, if both 'Stock Market' and 'Gold Returns' are affected similarly by the prevailing information.

iv) Modelling time varying correlation between Bitcoin and S&P 500's volatility through DCC GARCH.

v) Suggesting whether periods of optimism where stock market returns increase can cause investors to flock to Bitcoin through an ARDL model.

4. Hypotheses of the Study:

This paper aspires to study the relationship among variables through the use of following hypothesis:

Hypothesis 1:

H_0: The model is stable and features no break points.

H_1: The model is not stable and feature break points.

Hypothesis 2:

H_0: The variables feature no unit root.

H_1: The selected feature a unit root.

Hypothesis 3:

H_0: There exists no causality in granger sense among the variables.

H_1: There is a significant causality in granger sense among the variables.

Hypothesis 4:

H_0: There is no cointegrating relationship between the selected variables.

H_1: There is a significant cointegration between the selected variables.

5. Research Methodology:

5.1. Variables:

For the purposes of the study, daily prices of S&P 500 and Bitcoin were used. Bitcoin prices were converted to a 5-day trading week to maintain comparability with the Stock index. For all practical purposes, the price of Bitcoin before 2014 was inaccurately reported, or due to illiquidity, differed from exchange to exchange. Thus, to maintain proper real-world applicability, daily prices were taken from December 2014. The prices were reported by Coinbase - a major cryptocurrency exchange, S&P 500 prices were obtained from Federal Reserve Economic Data (FRED). For identification of structural breaks, weekly data was taken to ensure that model is not over fitted. This also ensures that the break periods obtained were significant structural changes and not minor in nature.

5.2. Unit Roots Test:

Stationarity means that the statistical properties (i.e. mean, variance and autocorrelation structure etc.) of a process generating a time series remains constant over time. Testing for stationarity is a must before proceeding to statistical methods because many methods often strongly depend on this particular assumption.

Augmented Dickey-Fuller (ADF), Philips-Perron (PP) and Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test are most used to check the stationarity of the variables.

5.2.1. Augmented Dickey Fuller (ADF) Test:

ADF test expands the Dicky Fuller Test, improving the methodology by controlling for autocorrelation. The Augmented Dickey-Fuller (ADF) test forms a parametric correction for higher-order correlation by surmising that the **y** series sticks to an AR (1) process. ADF is broadly applied because its critical values are not unstable.

The hypotheses are as follows:

Null Hypothesis (H_0): Presence of Unit Roots.
Alternate Hypothesis (H_1): Absence of Unit Roots.

In case the null hypothesis is rejected, then the series is stationary.

$$\Delta X_t = a_0 + a_1 T + \beta X_{t-1} + \sum_{j=1}^{p} \delta_j \Delta X_{t-j} + \varepsilon_t \qquad \ldots\ldots\ldots\ldots\ldots..(1)$$

Here, the first term denotes a constant, the second is representative of the trend's coefficient and **j** forms the lag order of the autoregressive process.

5.2.2. Phillips Perron (PP):

The Phillips-Perron (PP) test suggests a method controlling for both serial autocorrelation and heteroscedasticity in errors which makes it distinct from the ADF test.

The PP test modifies the existent test so that the distribution of the test statistic is not disturbed by serial correlation.

The PP test formulation can be written as-

$$\zeta_\alpha = \zeta_\alpha(\frac{\gamma_0}{f_0})^{1/2} - \frac{T(f_0 - \gamma_0)(se(\widehat{\alpha}))}{2f_0^{\frac{1}{2}}s} \quad \ldots\ldots\ldots\ldots\ldots..(2)$$

5.2.3. Kwiatkowski, Phillips, Schmidt, and Shin (KPSS) Test:

KPSS reverses the null and alternate hypothesis of the previous tests. Previously, if presence of unit root is not rejected, it signified the integration of the series at order one. KPSS's novelty lies in the fact that the test checks for unit root both about mean and a linear trend. By dividing a series into a deterministic trend (β_t), a random walk (r_t), and a stationary error (ε_t), KPSS also allows testing for stationarity within a deterministic trend.

Xt = rt + βt + εt $\ldots\ldots\ldots\ldots\ldots..(3)$

LM statistic of the hypotheses states that the random walk has zero variance and is defined by the equation:

$$LM = \sum_t \frac{S(t)^2}{T^2 f_0} \quad \ldots\ldots\ldots\ldots\ldots..(4)$$

$$S(t) = \sum_{r=1}^{t} \mu_r \quad \ldots\ldots\ldots\ldots\ldots..(5)$$

KPSS test uses the residuals from a OLS regression given by

$$y_t = x_t'\delta + \mu_t \quad \ldots\ldots\ldots\ldots\ldots..(6)$$

5.3. CUSUM Test:

The CUSUM test (Durbin, Brown, Evans 1975) is a sequential analysis technique stability test based on the multiple linear regression model of the form-

$$\widehat{Y} = b_0 + b_1X_1 + b_2X_2 + \cdots b_pX_p \quad \ldots\ldots\ldots\ldots\ldots.(7)$$

Parameter instability is found if Cumulative Sum goes beyond the two critical lines. Values of the sequence beyond a certain expected range propound a structural change in the model over time. Instability in the model is denoted when residuals go beyond the standard error band.

$$W_t = \sum_{r=k+1}^{t} \frac{W_r}{S} \quad \ldots\ldots\ldots\ldots\ldots(8)$$

For t=k+1……. T, where W_t is the recursive residual and S is the standard error pertaining to the recursive regression. The movement of W_t beyond the Confidence interval bands signify instability. CUSUM test is used to vet Bitcoin and S&P 500 returns coefficients stability.

5.3.1. CUSUM Square Test:

The CUSUM of Squares Test (Durbin, Brown, Evans 1975) hinges on the test statistic:

$$\mathbf{S_t} = \left(\sum_{\mathbf{r=k+1}}^{\mathbf{t}} \mathbf{w_r^2}\right) / \left(\sum_{\mathbf{r=k+1}}^{\mathbf{t}} \mathbf{w_r^2}\right) \qquad \ldots\ldots\ldots\ldots\ldots..(9)$$

The anticipated value of S_t under the hypothesis of parameter constancy is:

$$\boldsymbol{E(S_t)} = \frac{\boldsymbol{(t-k)}}{\boldsymbol{(T-k)}} \qquad \ldots\ldots\ldots\ldots\ldots..(10)$$

5.4. Granger Causality

In order to find out whether there are short-run causality exists between S&P 500 returns and Bitcoin returns, the linear granger causality test is applied. Causality in granger sense is established when one variable improves the predictability of another variable. A variable X has a causal relationship with variable Y if X is the cause of Y or vice-versa. However, with granger causality, we don't test a true cause-and-effect relationship; What we want to know is if a specific variable comes always precedes the other by a constant lag. Granger causality is usually checked for linear regression models.

$$\begin{aligned} y_t &= \alpha_0 + \alpha_1 y_{t-1} + \ldots. + \alpha_l y_{t-l} + \beta_1 x_{t-1} + \ldots. + \beta_l x_{t-l} + \epsilon_t \\ x_t &= \alpha_0 + \alpha_1 x_{t-1} + \ldots. + \alpha_l x_{t-l} + \beta_1 y_{t-1} + \ldots. + \beta_l y_{t-l} + \mu_t \end{aligned} \qquad \ldots\ldots\ldots\ldots\ldots(11)$$

where; L refers to the number of lagged observations in the model.
ε_t and u_t are residuals (prediction errors) for each time series.
α and β are the coefficients of the respective variables.

If the coefficient of S&P 500 returns is statistically different from zero for different lags then we fail to reject the absence of granger causality and we can say that Bitcoin returns granger causes S&P 500 returns. Similarly, if the coefficient of Bitcoin returns is statistically significant then the direction of causality is from S&P 500 returns to Bitcoin returns. If coefficients of both S&P 500 returns and Bitcoin returns are different from zero then we can say that there exists bidirectional causality, therefore concluding that both S&P 500 returns and Bitcoin returns cause each other.

5.5. Johansen Cointegration Test:

The co-integration of two series makes the error term stationary and the regular ordinary least square regression of Y on X is justifiable which results in long-term equilibrium between two variables. The Engle-Granger (1987) test stated that a linear combination of two or more than two non-stationary series possibly be stationary. Johansen's methodology starts by taking the Vector Autoregression (VAR) of order p which is given by -

$$y_t = \mu + A_1 y_{t-1} \ldots\ldots + A_p y_{t-p} + \epsilon_t$$ …………… (12)

This VAR can be re-written as-

$$\Delta y_t = \mu + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-1} + \epsilon_t$$

$$where$$ …………… (13)

$$\Pi = \sum_{i-1}^{p} A_i - I \ and \ \Gamma_i = -\sum_{j=i+1}^{p} A_j$$

Johansen proposes 2 distinct tests: the maximum eigenvalue or lambda max test and the trace test

$$J_{trace} = -T \sum_{i=r+1}^{n} ln(1-\lambda_i)$$

$$J_{max} = -T \ ln(1-\lambda_{r+1})$$ …………… (14)

The test of cointegration is based on finding whether there is a stationary combination of non-stationary variables.

5.6. Impulse Response Function

VIRF (Variable Impulse Response Function) provides a new method to enable one to check the effect of shock on one variable on the other. IRF checks the effect of a onetime shock on the current and future value of the dependent variables. In the latter case, there is no difficulty in considering 'realistic' as they can be drawn from the approximated distribution of the innovations.

It can be represented as $\boldsymbol{u_t} = \boldsymbol{P_{\epsilon_t}} \sim (\mathbf{0}, \boldsymbol{D})$

Where D is the diagonal matrix.

5.7. Dynamic Conditional Correlation Generalized Autoregressive Conditional Heteroscedasticity (DCC) GARCH

Unlike constant conditional correlation or weighted models, DCC GARCH proves to be better suited due to its time varying nature. The model can allow one to demonstrate changes in correlation of volatilities of two or more assets over a period. In our study, testing for increase or decrease in correlation in volatilities can allow us to ascertain whether same information affects the changes in returns of both the asset.

The model can be represented as -

Ht = DtRtDt The conditional variance is calculated using the GARCH (1.1) methodology

$$h_{it} = \omega_i + \sum_{x=1}^{X_i} \alpha_{ix} r_{it-x}^2 + \sum_{y=1}^{Y_i} \beta_{iy} h_{it-y}, \ for \ i = 1, 2, \ldots\ldots, k$$ …………… (15)

Here, ωi, αix, and βiy are non-negative and $\sum Xi\ \alpha ix + \sum Yi\ \beta iy < 1$; αix is the short-run persistence x=1 y=1 of the shocks to returns Y to long-run persistence and the number of assets is denoted by k.

The conditional correlation can be represented as:

$$\rho_{ij,t} = \frac{(1-a-b)\overline{Q} + a\varepsilon_{t-1} - 1\varepsilon'_{t-1} + bQ_{t-1}}{\sqrt{(1-a-b)\overline{Q} + a\varepsilon_{t-1} - 1\varepsilon'_{t-1} + bQ_{t-1}}\sqrt{(1-a-b)\overline{Q} + a\varepsilon_{t-1} - 1\varepsilon'_{t-1} + bQ_{t-1}}} \quad \text{................(16)}$$

5.8. Bai Perron Multiple Break-Point Test:

Tests for instability in regression models was studied first by Chow (1960). However, unlike Chow's test. which requires break dates to be known, Bai-perron's test enables one to find multiple breaks without a priori knowledge of break dates. Thus, estimation of break dates serves as Bai-Perron's primary usage, we consider the following multiple linear regression with m breaks (m + 1 regimes):

$$\boldsymbol{y_t = x'_t\beta + z'_t\delta_j + u_t \qquad where, t = T_{j-1} + 1 \dots \dots \dots T_j} \quad \text{.............. (17)}$$

6. Data Analysis and Interpretation:

Table-2 presents the descriptive statistics of Bitcoin and S&P 500 prices. It is observed that S.D of Bitcoin prices far exceed the S&P 500 prices. A high skewness is an indication that prices are not normally distributed.

Table-1: Variables, Source of Collection, Periodicity and Frequency

Name of the Variable	Source of Collection	Period and Frequency
Bitcoin (BITCOINUSD)	FRED. Bank of St. Louis	December 2014-September 2021. daily
Standard and Poor's 500	FRED. Bank of St. Louis	December 2014-September 2021. daily

Table-2: Descriptive Statistics of Bitcoin Price and S&P 500

Statistic	Bitcoin Prices ($)	SP500 Price ($)
Minimum	193.536	1850.274
Maximum	61175.850	4529.588
Range	60982.314	2679.314
1st quartile	615.634	2147.137
Median	5819.289	2675.106
3rd quartile	9759.054	3007.966
Sum	3283547.534	971941.289
Mean	9249.430	2737.863
Variance	177147458.241	422403.793

Standard deviation	13309.675	649.926
Skewness (Pearson)	2.283	0.937
Kurtosis (Pearson)	4.530	0.232
Mean Absolute Deviation	8521.269	509.414
Median Absolute Deviation	4974.944	474.390

Figure-1: Represents the Plot of Prices of S&P 500 and Bitcoin.

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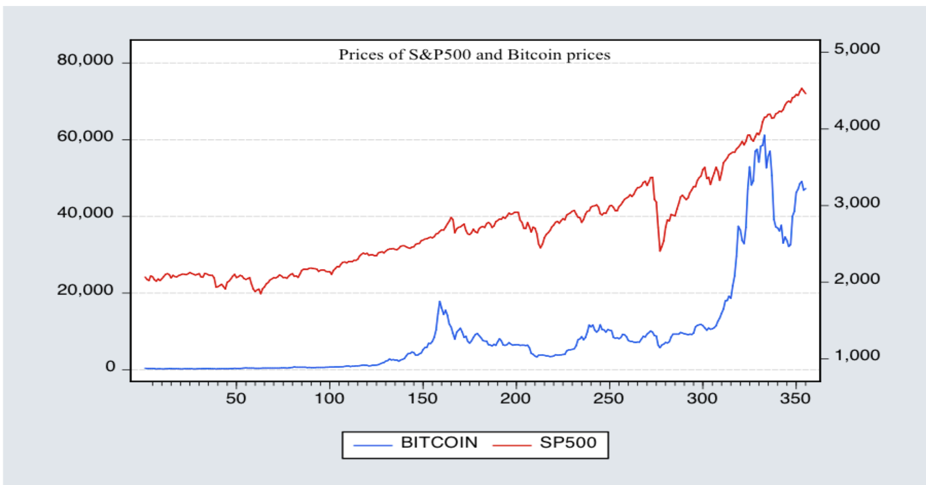
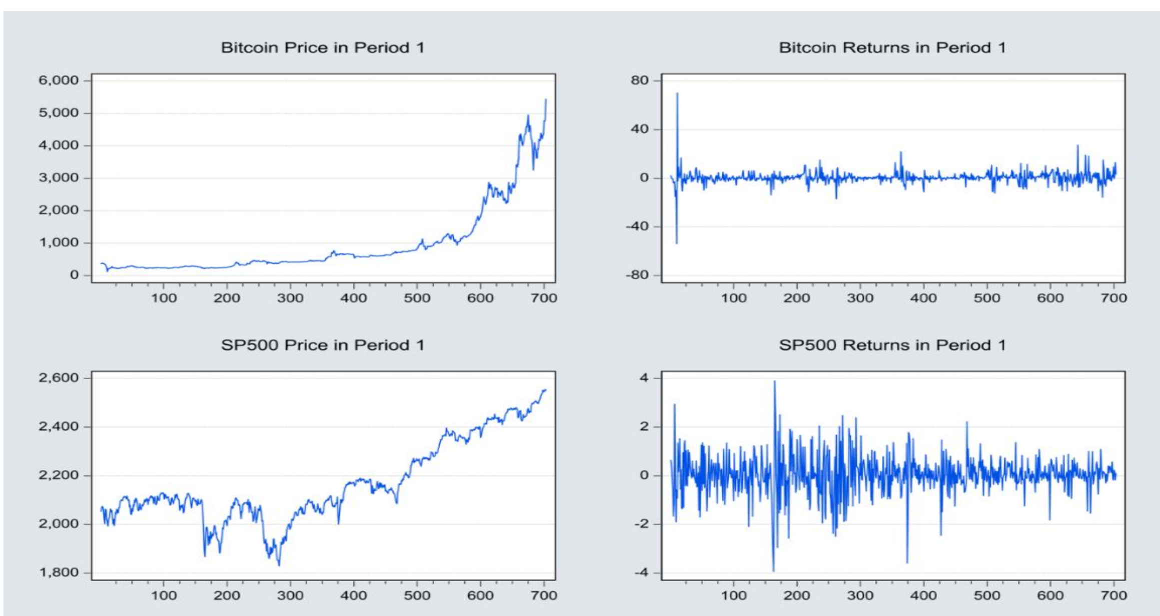
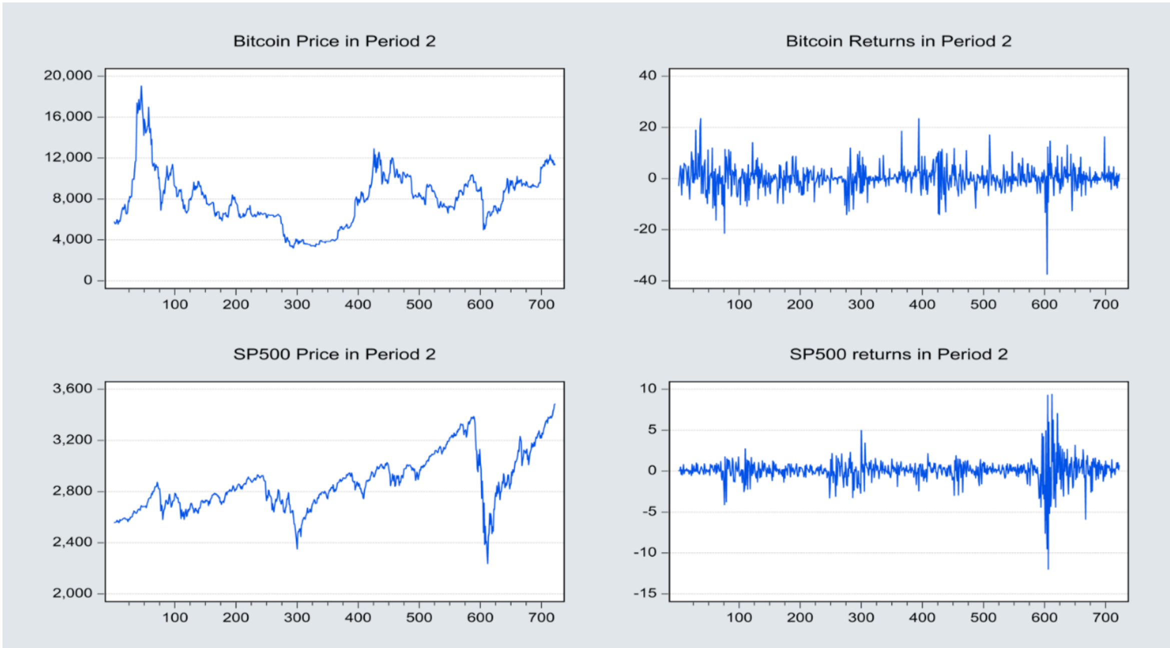






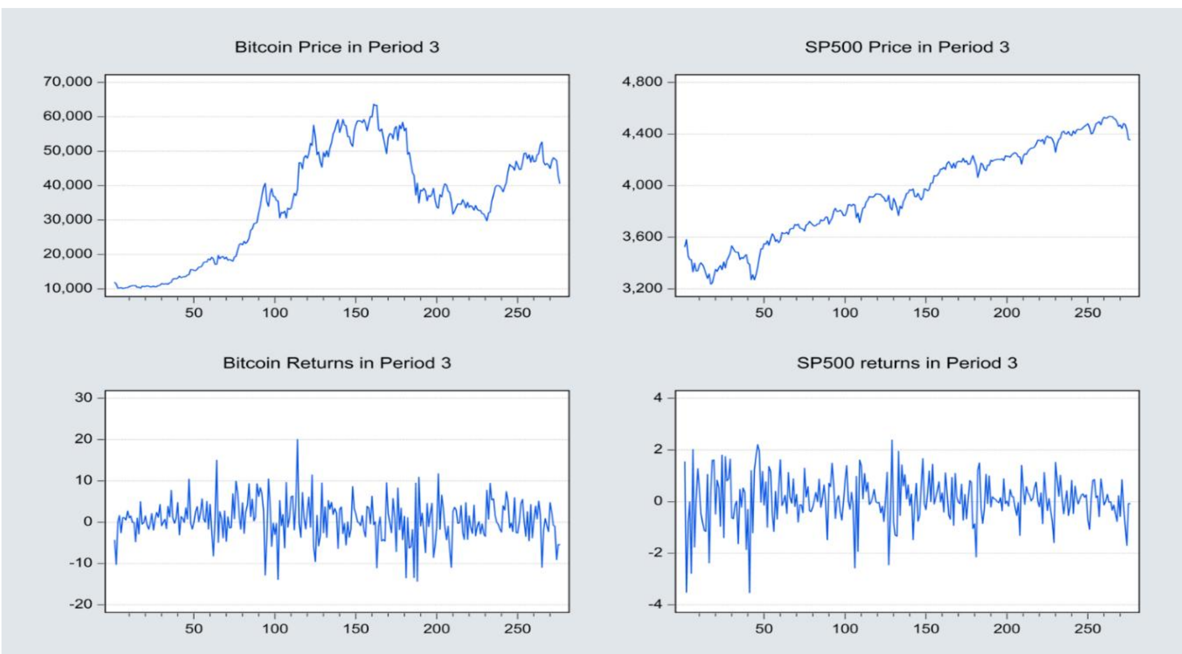
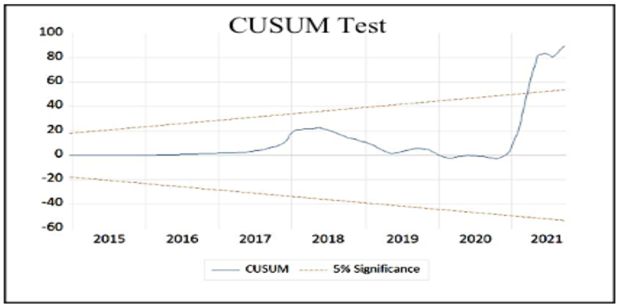
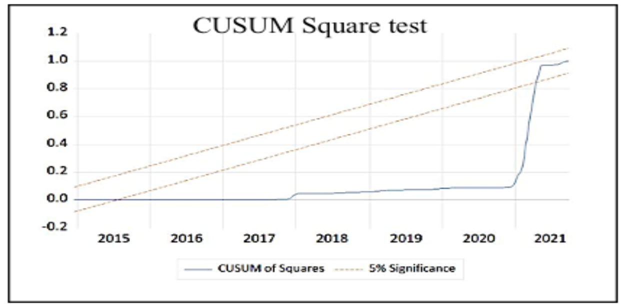


Figure-2 and 3 represents the CUSUM and CUSUM-square test to determine the stability of coefficients between Bitcoin and S&P 500. As noted, the test finds instability in the model as the total sum of the residuals extend beyond the 5% critical lines - this is emblematic of structural change.

Figure-2 and 3: CUSUM and CUSUM - Square Test for Establishing Instability in the BITCOIN - S&P 500 Model





Bai-Perron structural break test is presented in Table-3. After addition of dummy variable, CUSUM test indicated stability. Structural breaks were reported in 10/09/2017 and 9/21/2020.

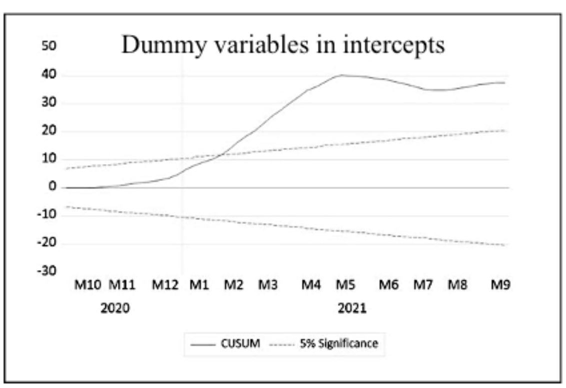
Table-3: Bai-Perron Multiple Breakpoint Test

Break Test	F-Statistic	Scaled F-Statistic	Critical Value
0 vs 1	156.1276	312.2552	11.47
1 vs 2	6.738159	13.47632	12.95
2 vs 3	5.010925	10.02185	14.03

Periods	Start Date	End Date
Period 1	09/12/2014	10/09/2017
Period 2	10/09/2017	09/21/2020
Period 3	09/21/2020	21/09/2021

In Figure-4, Dummy variables are used in intercepts in-order to remove instability. CUSUM with dummy variable in intercepts still indicates instability, thus, dummy variables are added to S&P 500 coefficient in Figure-5. This was observed to remove instability from the Bitcoin / S&P 500 model

Figure-4 and 5: CUSUM Test with Dummy in Intercepts and in Intercepts and Coefficients



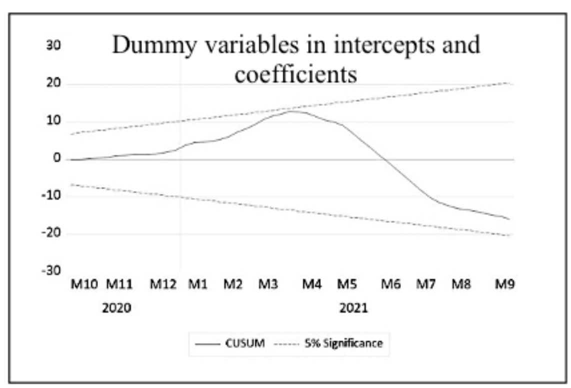


Table-4 presents the optimum VAR order selection for the three periods through the Aikake Information Criteria.

Table-4: Determination of Lags through AIC

No of Lags	Period 1	Period 2	Period 3
1	**14.437**	19.625	**22.152**
2	14.445	19.580	22.162
3	14.449	19.576	22.183
4	14.454	19.578	22.211
5	14.446	**19.537**	22.233

Table-5 and 6 presents ADF, PP and KPSS test for three periods. Null-Hypothesis in case of ADF and PP tests represents non-Stationarity. The observed value exceeds the critical value in 5% confidence level. Thus, the null hypothesis of unit root presence cannot be rejected. Contrarily, in KPSS tests, the null hypothesis is of stationarity. In all periods, null hypothesis is rejected.

Table-5: ADF, PP and KPSS test for Bitcoin Price

Test Result	Period 1	Period 2	Period 3
ADF	Non-Stationary Null Hypothesis accepted from P value	Non-Stationary Null Hypothesis accepted from P value	Non-Stationary Null Hypothesis accepted from P value
PP	Non-Stationary Null Hypothesis accepted from P value	Non-Stationary Null Hypothesis accepted from P value	Non-Stationary Null Hypothesis accepted from P value
KPSS	Non-Stationary Null Hypothesis is rejected from P value	Non-Stationary Null Hypothesis is rejected from P value	Non-Stationary Null Hypothesis is rejected from P value

Table-6: ADF, PP and KPSS Test for S&P 500 Price

Test Result	Period 1	Period 2	Period 3
ADF	Non-Stationary Null Hypothesis accepted from P value	Non-Stationary Null Hypothesis accepted from P value	Non-Stationary Null Hypothesis accepted from P value
PP	Non-Stationary Null Hypothesis accepted from P value	Non-Stationary Null Hypothesis accepted from P value	Non-Stationary Null Hypothesis accepted from P value
KPSS	Non-Stationary Null Hypothesis is rejected from P value	Non-Stationary Null Hypothesis is rejected from P value	Non-Stationary Null Hypothesis is rejected from P value

PP, ADF and KPSS tests are complimentary, yet serve as robust check to each other. The result revealed non-stationarity in level prices of S&P 500 and Bitcoin prices, enabling us to go ahead with the Cointegration test. Table-7 represents models of Johansen's Cointegration Test for period 1.2 and 3 respectively. As noted, cointegrating relation was found in Period 1 in both "Trace and Lambda max tests".

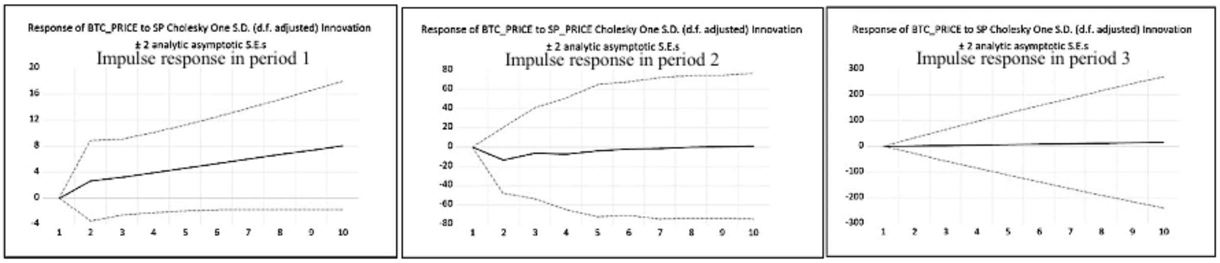
Table-7: Johansen Cointegration Test

Test	Number of Cointegrating Relationship in Period 1	Number of Cointegrating Relationship in Period 2	Number of Cointegrating Relationship in Period 3
Lambda max test	1	0	0
Trace test	1	0	0

The study found cointegrating relationship in period 1. Both maximum eigenvalue and trace value statistic suggested presence of one-cointegrating vector. Having found cointegrating relationship. one can make a conclusion that at least one market could've been used for predicting the other in the long run in Period 1 but not in period 2 and 3. Yet, this does not provide us with conclusive evidence of flow of information from SP500 to bitcoin in the short run. For that purpose, we carried out an Impulse Response Function. Effect of one S.D shock was seen on three periods. Initially, in every period. the immediate response is 0. In study period 1, Bitcoin prices responded to external shock on the system by displaying increasing response over the entire period which did not dissipate. However, sharp change is noticed in study period 2, where one S.D shock to S&P 500 causes Bitcoin prices to display a negative response with the effect dissipating by Time-interval 6. In study period 3, Bitcoin displays no response to S.D. in S&P 500 prices. The above finding, when studied in conjunction with the cointegration tests can prove to be significant. This change can be attributable to decoupling of bitcoin from the stock market. The suspected causes of why this might be happening are varied. The primary reasoning seems to be the increase in institutional flow of money into bitcoin for speculation. With crypto becoming too big to ignore. fund managers have been bullish on Bitcoin despite its high volatility. Institutional flows of money can distort the relationship due to heavy demand and inflow of money despite no apparent causation, such as periods of excessive optimism in the stock market. Another reasoning can be the emergence of Bitcoin Exchange traded funds. This can explain the de-linkage in study period 3 where bitcoin ETF's have raised optimism in the crypto market due to increase in adoption. On the similar lines, adoption by countries, most notably El Salvador. can induce similar responses.

Figure-6, 7 and 8 represent the impulse response function of BITCOIN to one unit of S.D shock to S&P 500. Figure-6 shows that impulse response of BITCOIN to S&P 500 is zero in first period. thereon increasing progressively. Impulse response to S&P 500 in period 2 is negative at first. returning to 0, indicating no long run persistence in response to shocks in S&P 500. In period 3, Bitcoin did not exhibit substantial response to S&P 500 shocks.

Figure-6, 7 and 8: Response of BITCOIN Prices to Shocks in S&P 500



To enable us to further fortify our assumptions. we conducted a VAR based Ganger causality test between 'Bitcoin returns' and 'S&P 500 returns'. For the purposes of the test. we conducted a stationarity test through ADF and PP. On finding stationarity, Optimal lag length of VAR model was determined through AIC.

Table-8 presents the Granger causality tests between 'S&P 500 returns' and 'Bitcoin returns'. For the purposes of the rests. Log transforms of the returns were taken. Granger causality from 'S&P 500 returns' to 'BITCOIN returns' was exhibited in the first period. No such causal relationships were observed in period 2 and 3 respectively. VAR order was determined through AIC.

Table-8: Did S&P 500 Returns Granger Cause Bitcoin Returns

Number of Lags	Period 1	Period 2	Period 3
1	**-15.491**	-14.511	**-15.561**
2	-15.487	-14.541	-15.549
3	-15.488	-14.545	-15.523
4	-15.482	**-14.551**	-15.499
5	-15.483	-14.545	-15.486

Test Statistic	Period 1	Period 2	Period 3
P value	0.022	0.804	0.296
Result	'Bitcoin returns' were **caused** in the granger sense by 'SP&500 returns'	'Bitcoin returns' were **not caused** in the granger sense by 'SP&500 returns'	'Bitcoin returns' were **not caused** in the granger sense by 'SP&500 returns'

Table-9 presents the F and Bounds test using an Autoregressive Distributed Lag Model to show the cointegration and causal relationship between 'S&P 500 returns' and 'Bitcoin Prices'. Due to the differing order of integration. we use an ARDL model. A relation can be symbolic that periods of optimism where S&P 500 returns rise can cause the Bitcoin prices to rise too. As noted from the F test statistic. only in the first period did we find long run equilibrium relationship between 'S&P 500 returns' and 'Bitcoin prices' - which can enable us to conclude that beyond the first period, periods of excessive jubilance or fear causing S&P 500 returns to spike or crash did not exhibit conjoint movement with Bitcoin prices.

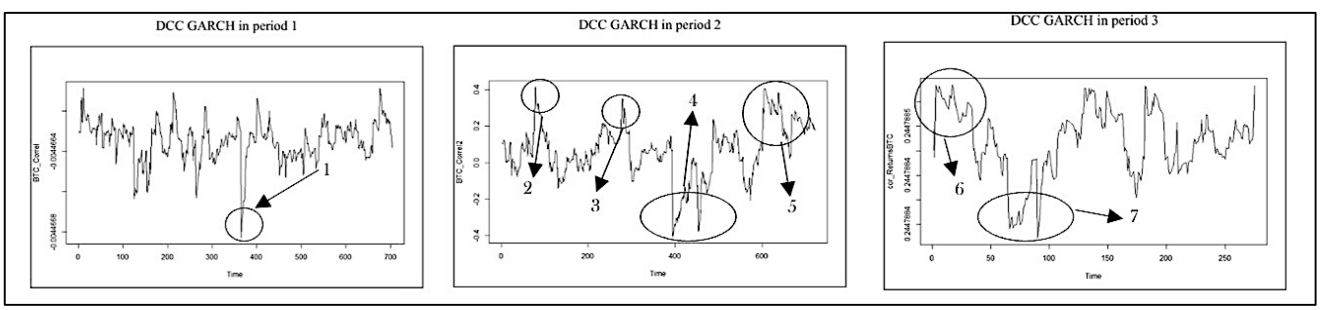
Table-9: ARDL based F test statistic to establish cointegration between variables of differing Order of Integration

Test Statistic	Period 1	Period 2	Period 3
F Value and I (0) Value at 5% confidence bands	F statistic > I(1) critical value at 1.5.10% intervals	F statistic < I(1) critical value at 1.5.10% intervals	F statistic < I(1) critical value at 1.5.10% intervals
Interpretation	Cointegration Exists	No Cointegration Exists	No Cointegration Exists

Figure-9 represents Dynamic Conditional Correlation GARCH Model to showcase correlation between the second difference (Fluctuations of Returns) of Bitcoin and SP500 prices. Volatility or the second difference of level prices is modelled by the Univariate GARCH model in Step-1 from Bitcoin and S&P returns. Progressively, a DCC model is applied to examine changes in correlation across time.

Figure-9: DCC GARCH Model to show Correlation among Volatilities

	Volatility(Second Difference) BTC-Price	Volatility(Second Difference) SP
Period 1		
Volatility(Second Difference) BTC-Price	1	**-0.004466**
Volatility(Second Difference) SP	**-0.004466**	1
Period 2		
Volatility(Second Difference) BTC-Price	1	**0.1794944**
Volatility(Second Difference) SP	**0.1794944**	1
Period 3		
Volatility(Second Difference) BTC-Price	1	**0.244788**
Volatility(Second Difference) SP	**0.244788**	1



Above figures represent the dynamic correlation of volatility over the entirety of each time period. This can represent the ‘strength’ and ‘direction’ of relationship ‘Bitcoin’ and ‘S&P 500 volatility’. The above matrix converges to a value eventually. This model provides better insight into changing relationships than Constant Conditional Correlation, which faces difficulty in representing instantaneous changes. Thus, the time varying nature of DCC-MGARCH can suit our purposes better.

	Period 1	Period 2	Period 3
Returns of BTC Alpha	0.24	0.12	0.06
Returns of BTC Beta	0.75	0.76	0.89
Returns of SP500 Alpha	0.2	0.27	0.27
Returns of SP500 Beta	0.71	0.72	0.6
α[DCC]	0	0.04	0
β[DCC]	0.92	0.92	0.93
Our results are consistent with those of theory. The coefficient α[DCC] is approximately 0 and sum of α[DCC]+β[DCC]< 0			

No	Suspected Cause	Period
1	China's Renminbi is devalued. Bitcoin is used as an instrument of Capital Flight or to transfer money. **Correlation of volatility falls as bitcoin is unable to affect stock markets**	Phenomenon is observed throughout 2016. Peak was in **May to June.2016**
2	US-China Trade war causes Stock markets to crash. Further causes can be tariff rises. Correlation of volatility spikes	**February to March.2018**
3	Crash in Stocks due to US-China trade wars **Volatility spikes due to conjunct movement in BITCOIN**	**December.2018**
4	Sharp rise in Bitcoin volatility due to Bitcoin Halving event. Traditional market remains unaffected; thus, correlation drops	**May. 2020**
5	COVID-19 led stock market crash. Bitcoin crashes- leads to rise in volatility correlation	**April. 2020**
6	2020 bull run commences. Bitcoin rises concomitantly with stocks. **Thus. both the asset's volatility rises.**	**September. 2021**
7	Sell off in cryptocurrencies by Bitcoin miners. Price falls by 15% in a day. Stock markets remain unaffected. **Correlation dips**	**January. 2021**

Conclusion:

The study is novel in its approach due to its focus on studying the changes in the Bitcoin - S&P 500 relation. While short and long run causal and integrating relationship has been previously studied. Accounting for structural breaks and volatility modelling is used for filling the gap in the existent literature. In addition, the results from the paper differ from that found in case of Tekinay & Kocakoc (2018)[9]. Moreover, through the paper, conclusive evidence was found which hinted towards decoupling of the Bitcoin and Stock market indices.

Initially, utilising the CUSUM test, we proved the presence of process changes. Bai perron multiple breakpoint test helped us divide the study period into three distinct segments. Unlike the established narrative, where, with increased participation, an asset class become more and more cointegrated with the traditional markets, our study found evidence of decoupling in the

[9] Tekinay, M. and Kocakoç, I.D. (2018). A Study of Relations Between Bitcoin, Currencies, Stock Exchanges and Commodities; *New Trends in Economics And Administrative Sciences, Izmir International Congress on Economic and Administrative Sciences*, December 2018.

bitcoin – S&P 500 relationship. This was emblemised by finding cointegration and causality in the granger sense in period 1 but not in period 2 and 3. The impulse response function similarly reveals Bitcoin's lack of response to shocks in S&P 500. Through an ARDL model. we were able to show that in the first period 2014: 2017, any increase in returns or decrease of S&P 500 returns caused conjoint movement in Bitcoin prices, emblemising that period of optimism or fear led to people flocking to bitcoin. Yet in period 2 and 3, no such phenomenon was found- which can indicate that people flocked to and shied away from bitcoin due to reasons entirely different from optimism or pessimism in the stock markets.

Modelling the correlation between volatilities revealed that bitcoin prices is heavily dependent upon events. We tried to explain the spikes and dips in correlation by studying the suspected events which might have resulted in both volatilities to increase. Very significantly, we found that certain events concerning bitcoin was unable to affect S&P 500's volatilities. This is marked by dips in the correlation. Yet, events affecting S&P 500 caused the volatilities to spike, which might indicate asymmetrical flow of information. Very simply. information affecting the stock market was able to affect Bitcoin, but such a conjecture was not true the other way round. Comparing the dynamic correlation over the three period also allowed us to establish the progressive increase in strength and direction of the volatility relationship. This can be explained due to either of the following reason. The first of the causes maybe excessive turbulence in the stock market. In the first period, the correlation seems to be weaker because 2014-2017 was marked with significant events which asymmetrically affected the Bitcoin market but not the stock market. Unlike the first period, 2017-2020 was marked by excessive turbulence in the traditional market. From trade wars to the covid crisis. bitcoin moved reacted heavily to sell offs in the S&P 500 market. The third period represents the 2020 bull-run after the covid crisis - which explains the high correlation. Another suspected cause might be the entry of amateur speculators, who reacted to sell offs or optimism in the stock market by exhibiting similar behaviour whilst trading Bitcoin?

This finding, however, might have little real-world applicability. This is because our study did not account for changing composition of the S&P 500 index. Secondly, this is based on historical data and might not be accurate for trading purposes. Similarly, Bitcoin's relationship with D_{xy}. Oil. Gold and Bonds may also be studied for a more robust model.

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